FAST AND BIJECTIVE RIGID DIGITIZED TRANSFORMATIONS

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Bijective approaches

- Digitized transformation that leaves invariant discrete lines (reflection) or discrete circles (rotation),
- Characterize rigid transformations that are bijective after digitization,
- Find a bijective map that minimizes an error with respect to the real rigid transformation.

Contributions

- New bijective approaches : CBDR, OTC, CDLR
- Experimental results

Considered metrics

- Average error

$$L_2(\mathsf{T},\alpha) := \sqrt{\frac{1}{\#D}\sum_{p\in D}\|\mathsf{T}(p)-\mathscr{R}_\alpha(p)\|^2}.$$

- Worst case error

$$L_\infty(\mathsf{T},\alpha) := \max_{p\in D}\|\mathsf{T}(p)-\mathscr{R}_\alpha(p)\|.$$

- Continuity error : with $N_8(p)$ the eight neighbour's of a pixel p

$$L_c(\mathsf{T}) := \sqrt{\frac{1}{8\#D}\sum_{p\in D}\sum_{q\in N_8(p)}\|\mathsf{T}(p)-\mathsf{T}(q)\|^2}.$$

- Combination of the worst case error and the continuity error

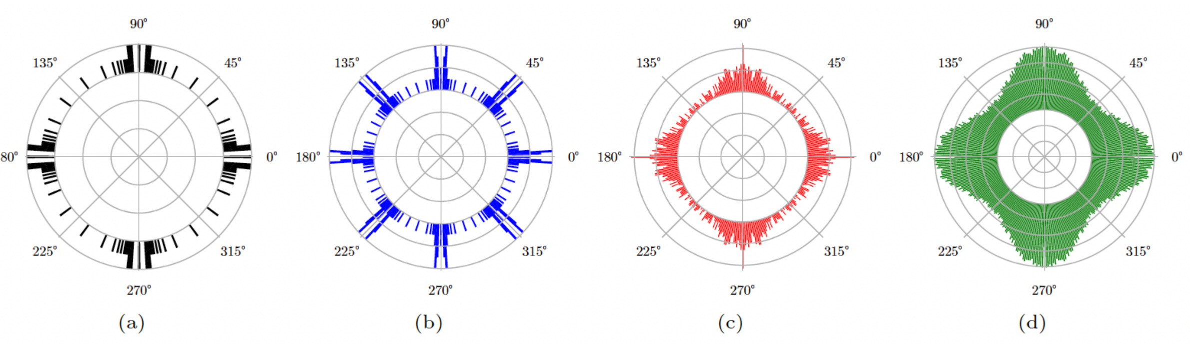
$$L_{\infty+\lambda c}(\mathsf{T},\alpha) := L_\infty(\mathsf{T},\alpha)+\lambda L_c(\mathsf{T}).$$

Composition of Bijective Digitised Reflections (CBDR)

- Bijective digitized reflections
- Construct the set of composition of bijective digitized reflections (computationally expensive)
- Remove duplicates
- Find the nearest composition of bijective reflection as

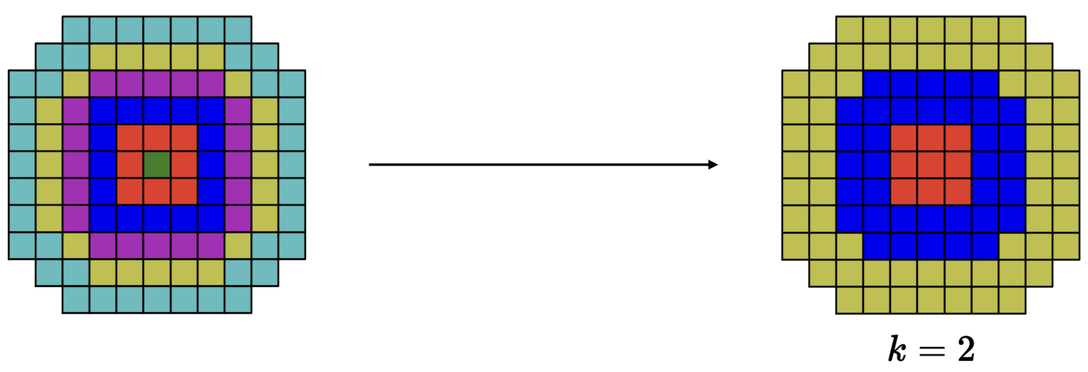
$$\bar{C}_\alpha^{k_{\max}} := \underset{\mathbf{m}\in C_\alpha^{k_{\max}}}{\arg\min}\ L_*(\Pi_{i=1}^4(\mathcal{D}\circ -\mathbf{m}_i\mathbf{pm}_i^{-1}),\alpha),$$

Resulting distribution:



Optimal Transport by Circle (OTC)

- Idea : Group concentric circles into k-tuples D^i

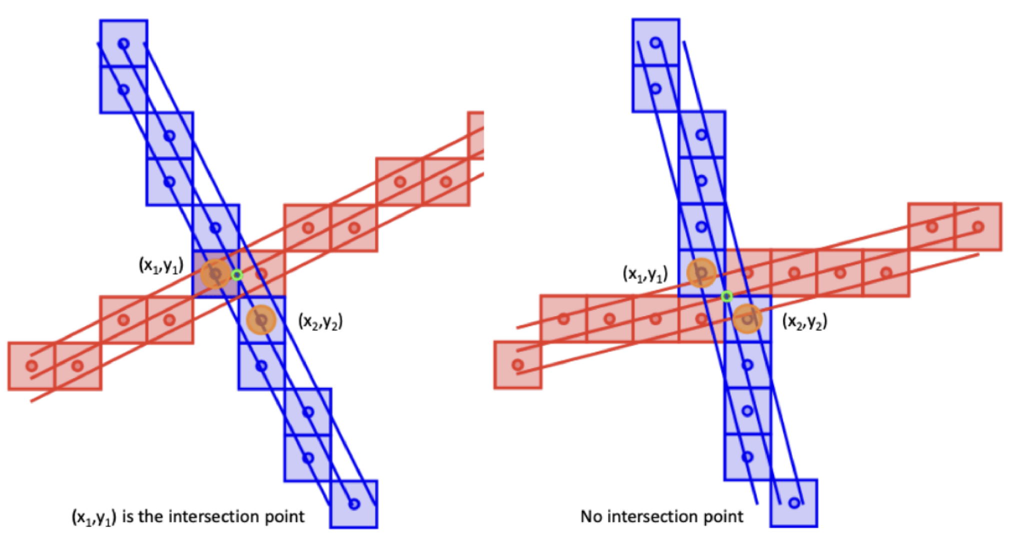


- For each D^i, compute the optimal assignment that minimizes

$$\left|\begin{array}{l}\mathbf{c}:\ \mathbb{Z}^2 \rightarrow \mathbb{R}^+\\ \quad \mathbf{D^i} \rightarrow \sum_{j,k}\|D^i_j - R_\alpha(D^i_k)\|_2^2\end{array}\right.$$

Composition of Discrete Line Reflections (CDLR)

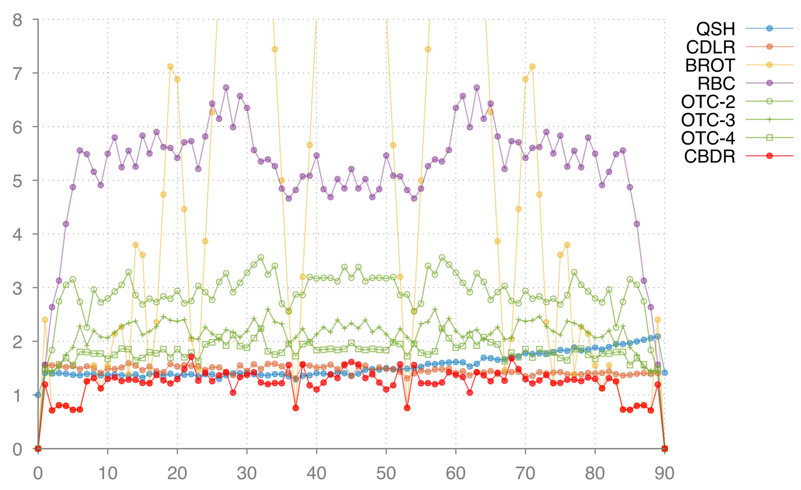
- Composition of 2 reflections
- choose the 2 reflections that minimises one of the metric error
- One-to-one mapping

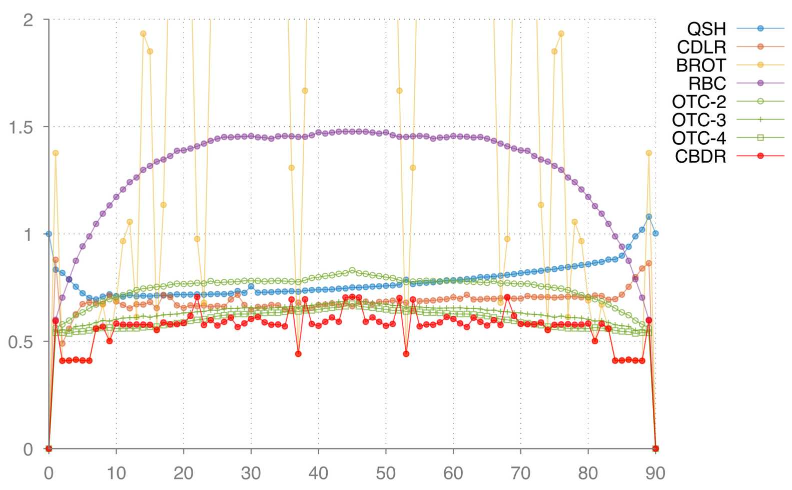


Time complexity and and image transformation time

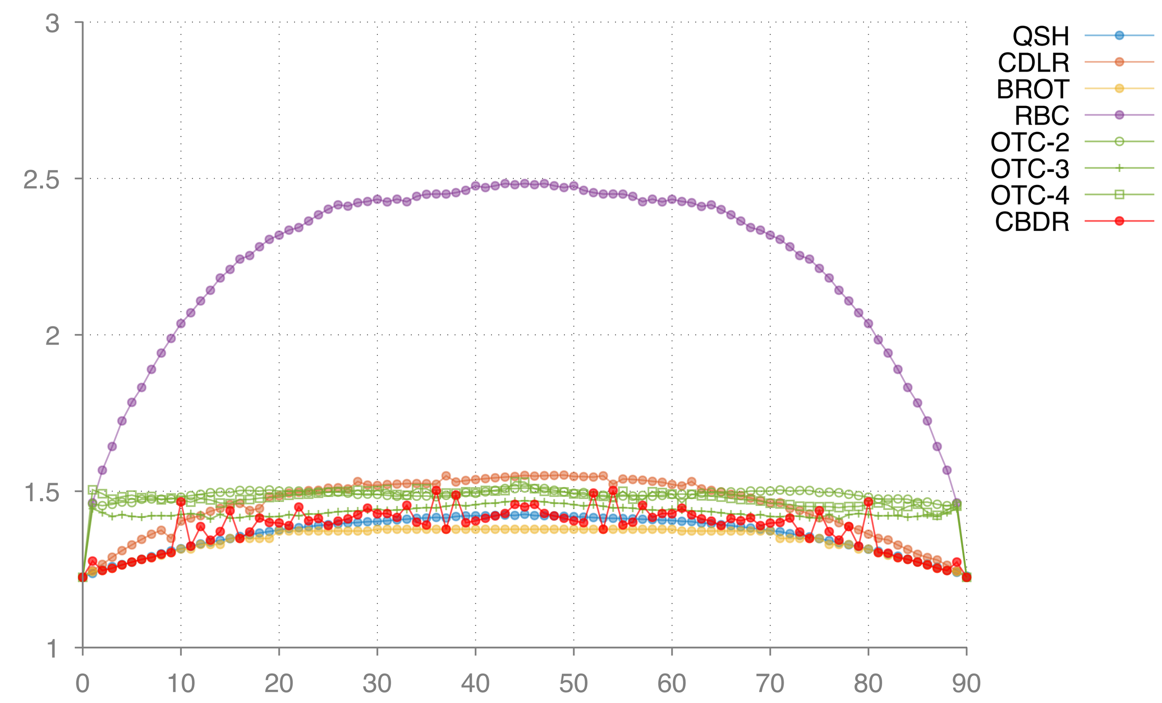
Method	QSH	CDLR	BROT	CBDR	RBC	OT	OTC-k		
Image transf.	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$	$O(N^8)$	$O(N^2\log(N))$		
Precomp.	n.a.	n.a.	n.a.	Eq.(11)	$O(N^2)$	n.a.	$O(k^3N^5)$		
							OTC-2	OTC-3	OTC-4
Image transf. (ms)	2.7	43.5	3.8	3.8	3.5	$> 10^5$	15.9	16	16
Precomp. (ms)	0	0	0	5300	13	0	$4\cdot 10^5$	$7\cdot 10^5$	$13\cdot 10^5$

Accuracy





Continuity



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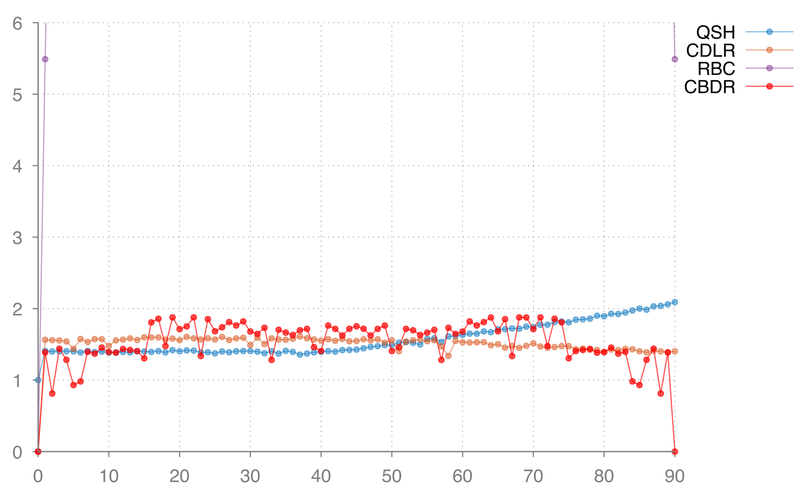
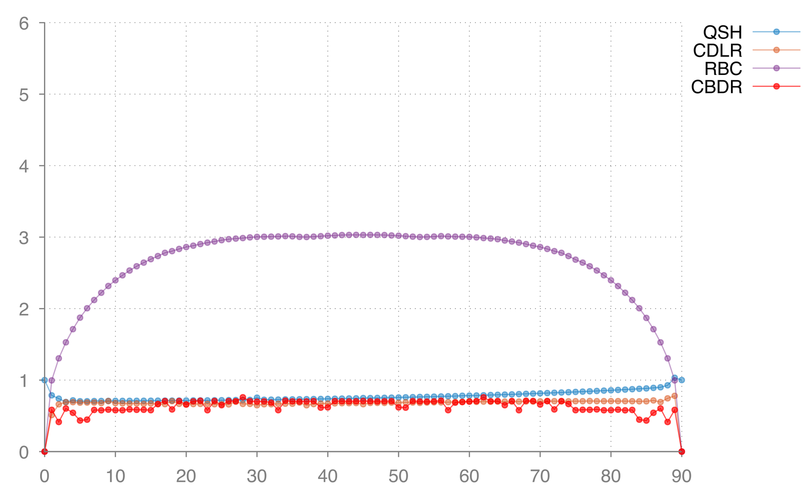
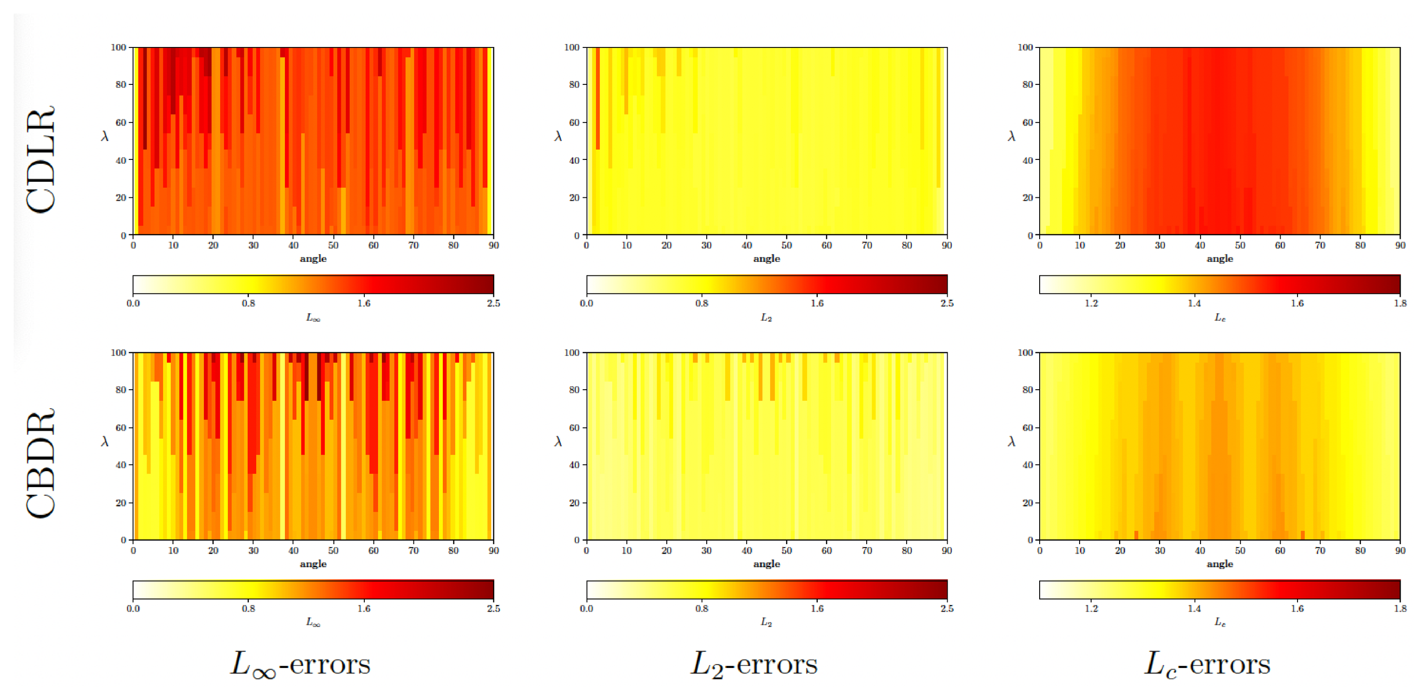




Optimization with respect to different metrics



Perspectives

- Tackling the problem of registration with bijective and as continuous as possible rigid transformation,
- Extending to 3D the proposed approximation methods.

Implementation

- All the presented approaches with quasi-shears are implemented in **DGtal**