A Parameter-Free Normal Vector Estimator on Digital Surfaces

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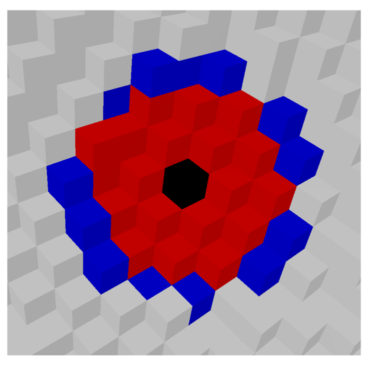
Context

Normal vector estimation methods on digital surfaces include those that use or adapt tools from Euclidean geometry ([1], [2], [5]), those that use digital planes and line segments ([3], [4]), or those that use convolution of trivial normals on surfels ([6]). We propose a parameter-free method that structures a neighborhood around a voxel into angular planar sectors.

Proposed method

Neighborhood around a voxel

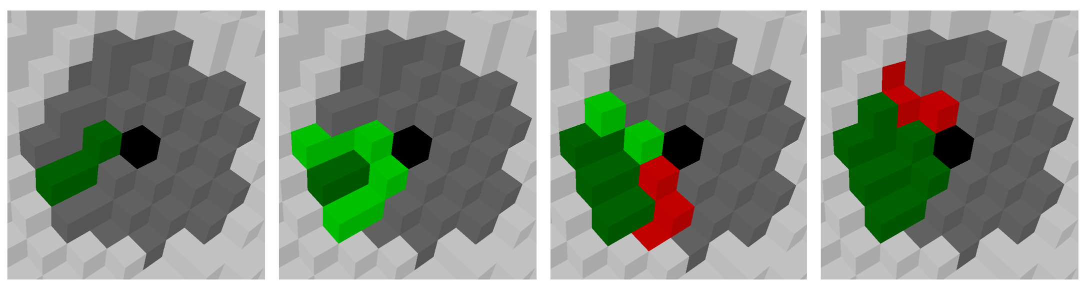
We compute the biggest radius r for which the neighborhood around a voxel on the surface is contained in a naïve digital plane (using the digital plane recognition algorithm from [8]), then consider the neighborhood of size $r+1$.



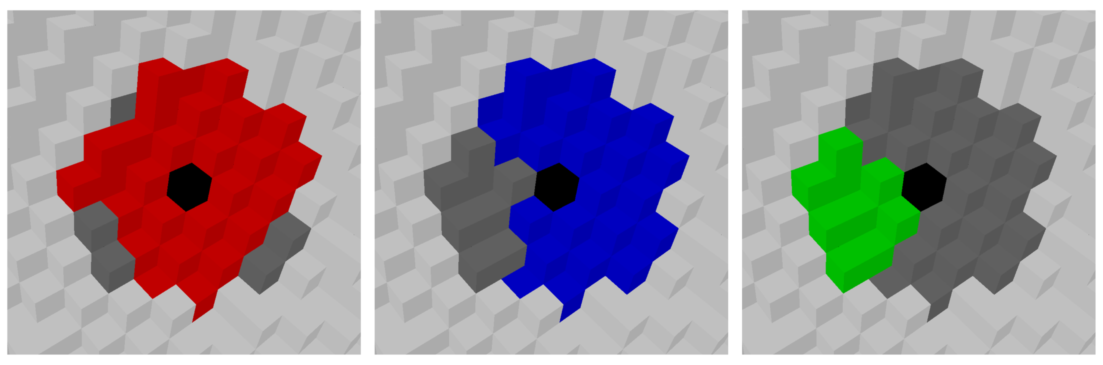
Some of the blue voxels do not belong to the red digital plane; they are thus significant for the determination of the surface's local structure, and serve as starting points for the construction of planar sectors.

Building planar sectors

A voxel path joins a border voxel to the center voxel; sectors are made by iteratively fusing adjacent voxel paths as long as the resulting sector is planar.



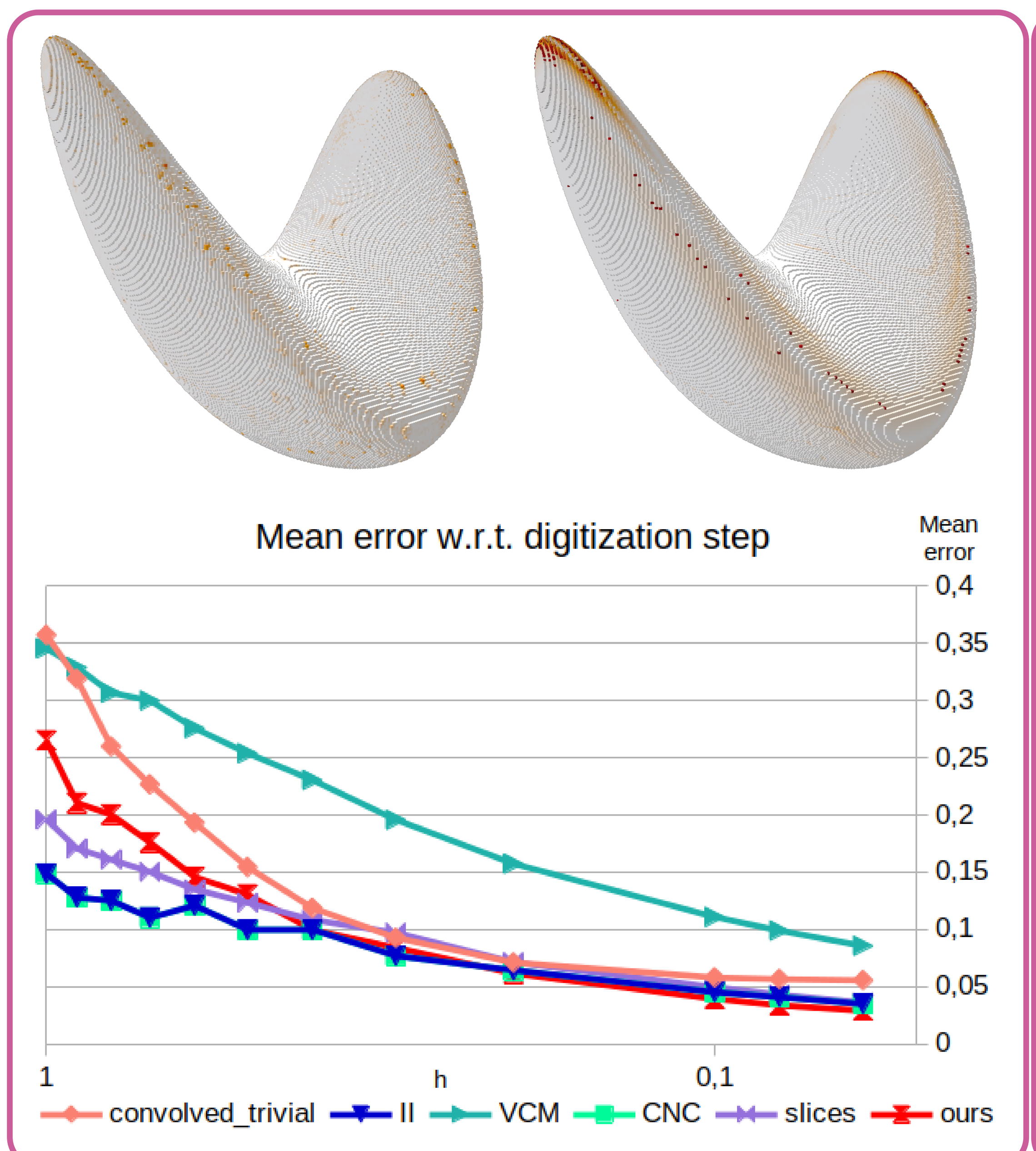
From planar sectors to normal vector

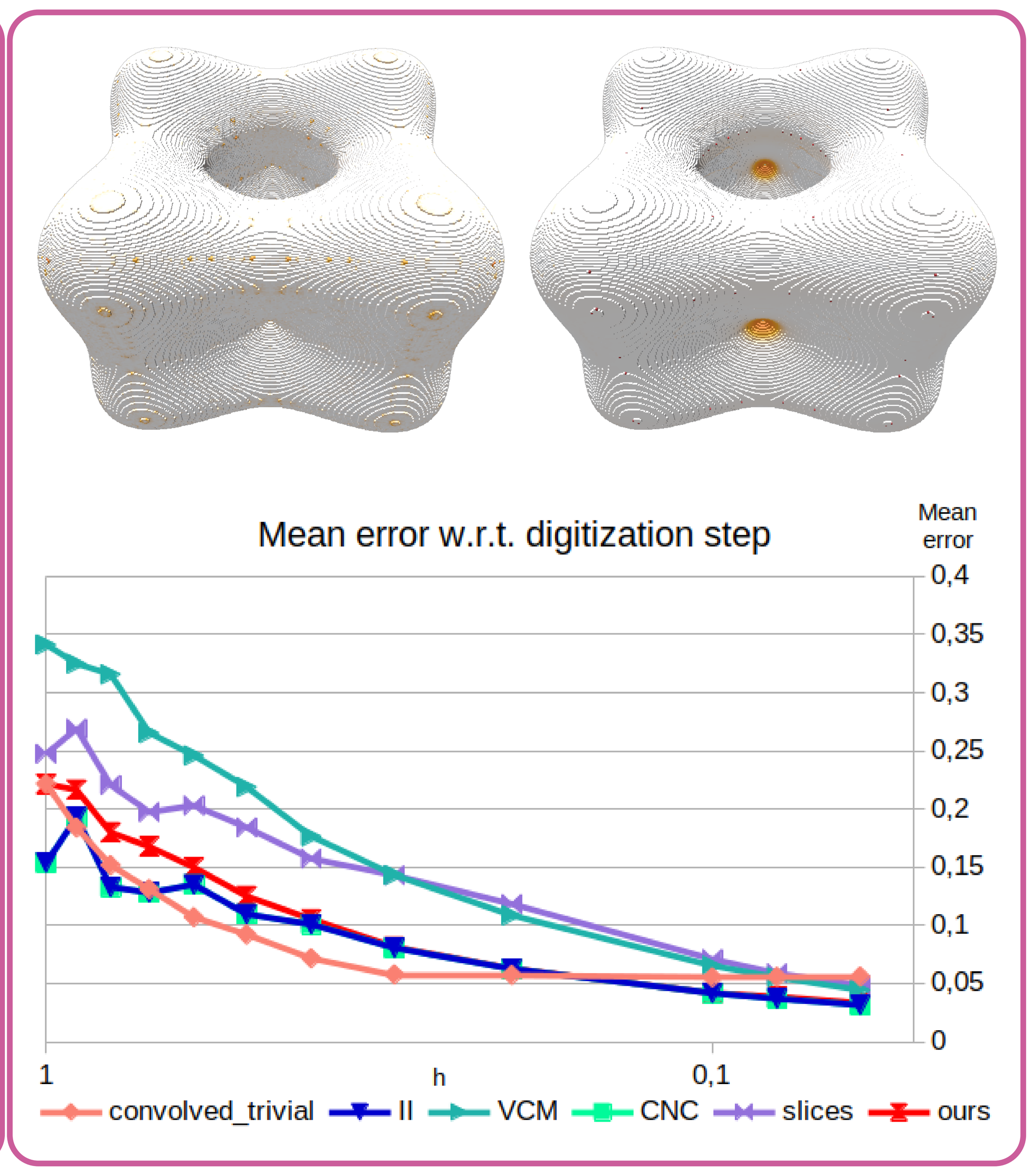


We use the area-preserving grid projection method in [7] to compute a fast approximation of the geodesic barycenter on the unit sphere of the normal vectors to the sectors' digital planes, weighted by their size in voxels.

Results

Experiments were conducted on digitizations of Euclidean shapes defined by implicit polynomials; the error measure is the angle deviation between the normal vector estimation at a voxel and the corresponding value on the Euclidean shape. Below, left is our method, right is Integral Invariants (II) [1]. The other methods on the graphs are convolved trivial normals [6], Voronoi Covariance Measure (VCM) [2], Corrected Normal Currents (CNC) [5], and slices [4].





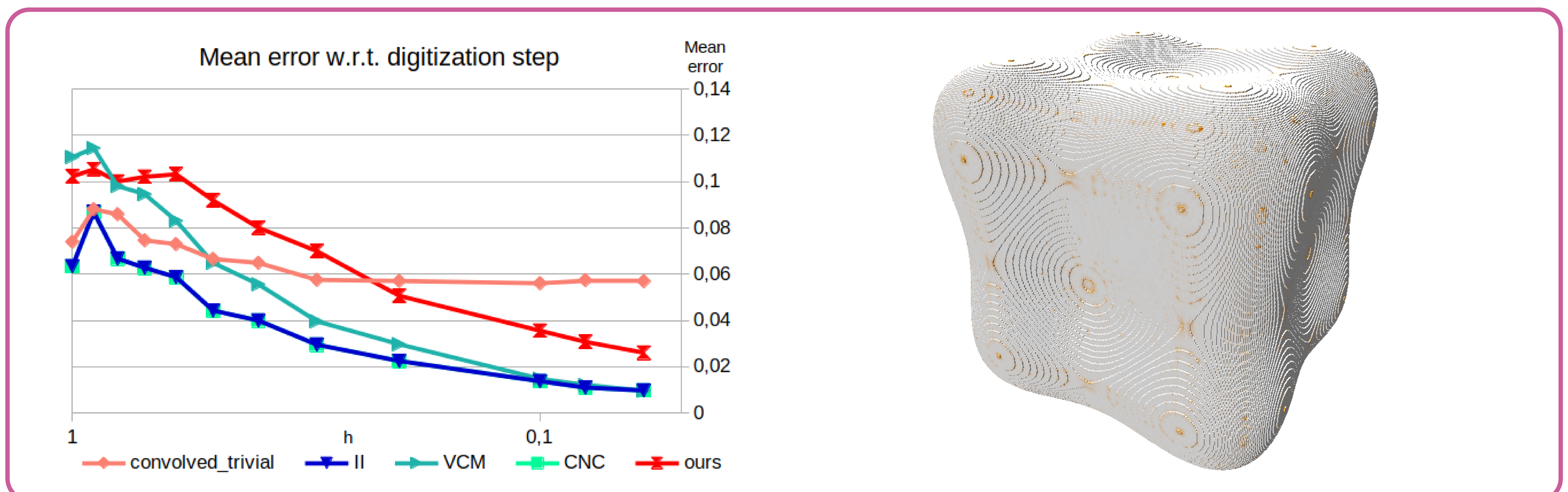


Table of max error for us and II on these examples (min is 0):

Method	(upper left)	(upper right)	(bottom)
Ours	0.739724	0.593599	0.645374
II	1.86961	1.10421	1.06705

Conclusion

- Our method is local, parameter-free, robust, and competitive with the state of the art regarding mean error and computation time. Its computation time can be greatly improved by setting a maximum radius for the neighborhood with minimal loss in precision.
- Some methods from the litterature have good mean errors on digital surfaces, but show decreased performance in specific conditions, which requires further investigation.

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