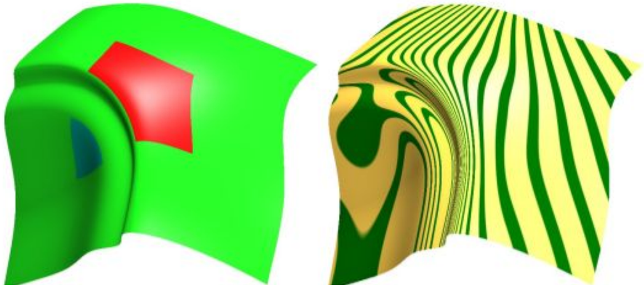
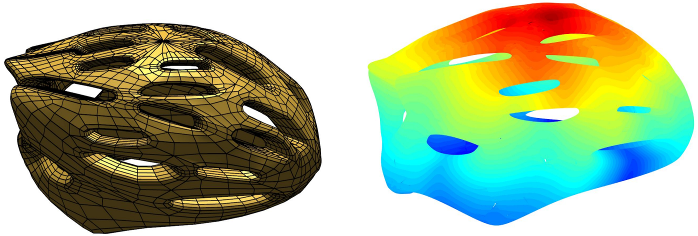
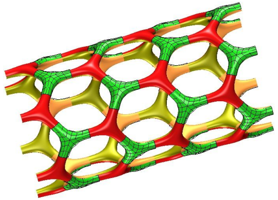
Polyhedral-net Splines for Geometry and Analysis







Jorg Peters U of Florida

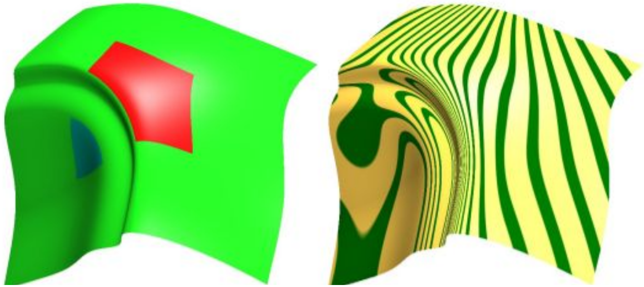
CIRM 2024

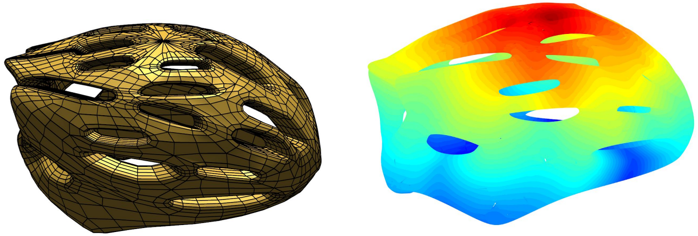
Joint work with
K Karciauskas,
Bhaskar Mishra
Seth Barber

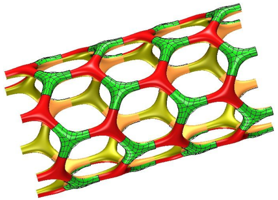
https://www.cise.ufl.edu/~jorg/

Polyhedral-net Splines for **Geometry** and Analysis

Jorg Peters







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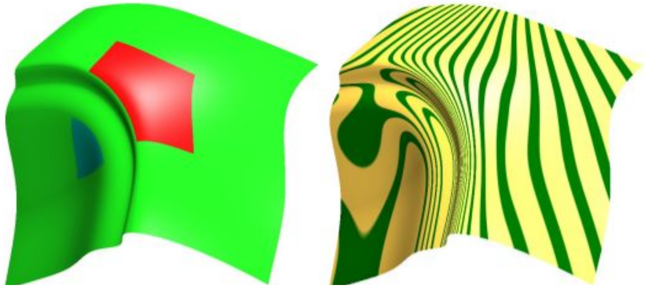
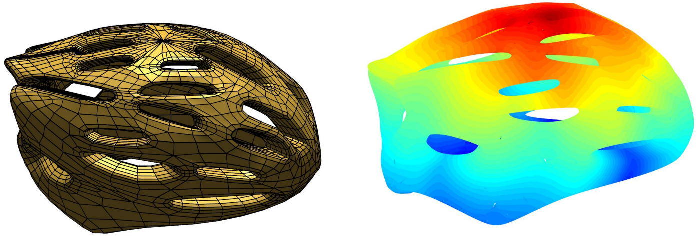
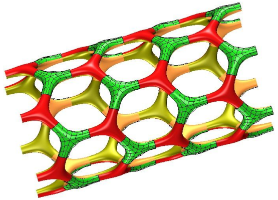
Polyhedral-net Splines for Geometry and **Analysis**

Jorg Peters







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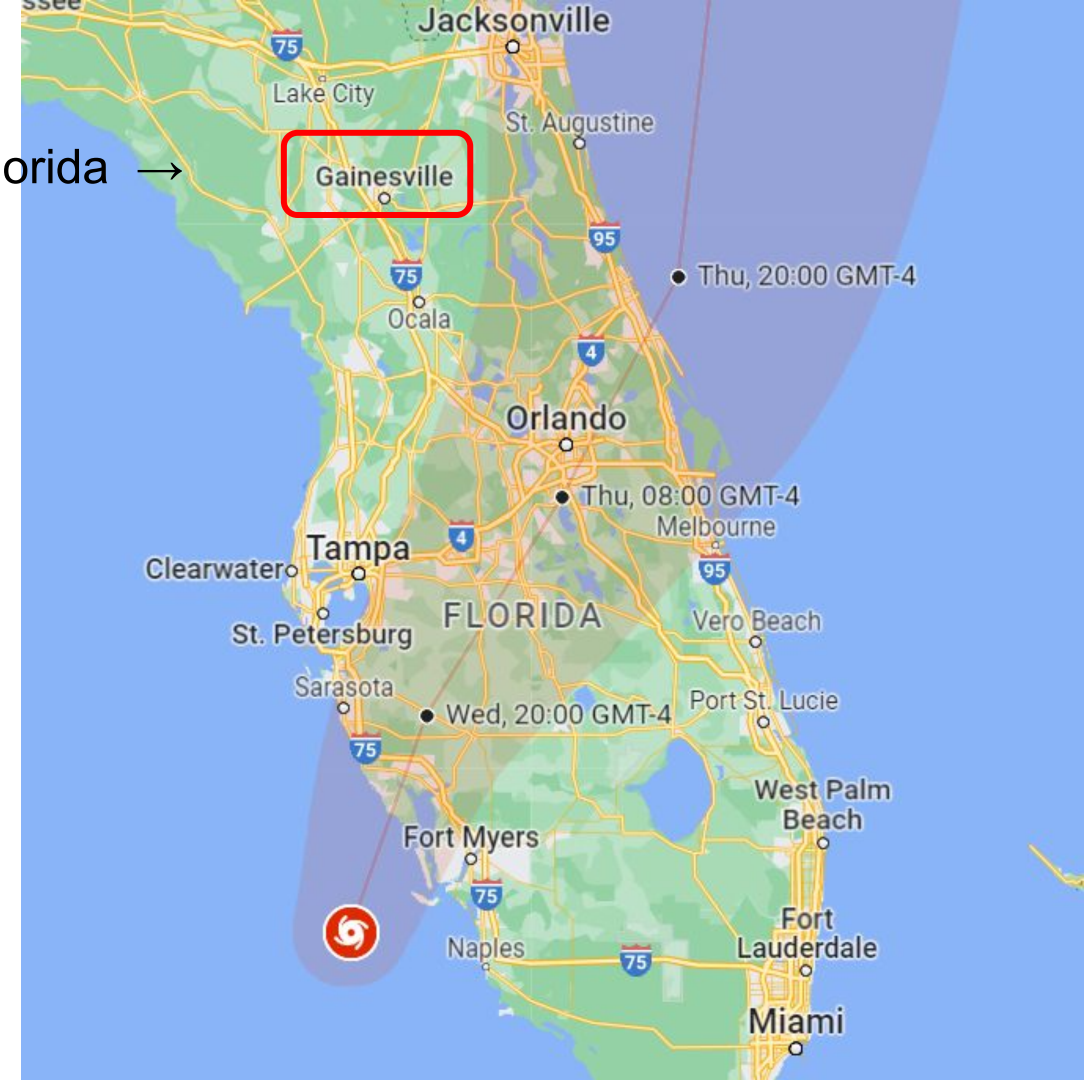
Jacksonville
Lake City
St. Augustine
Gainesville
Thu, 20:00 GMT-4
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Orlando
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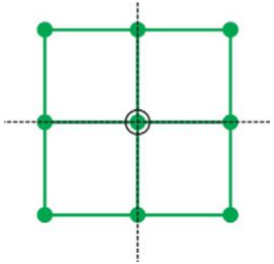
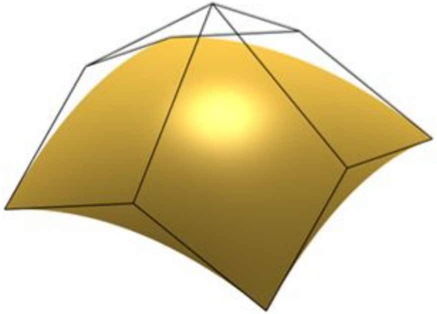
Melbourne
Tampa
Clearwater
FLORIDA
St. Petersburg
Vero Beach
Sarasota
Wed, 20:00 GMT-4
Port St. Lucie
West Palm Beach
Fort Myers
Naples
Fort Lauderdale
Miami

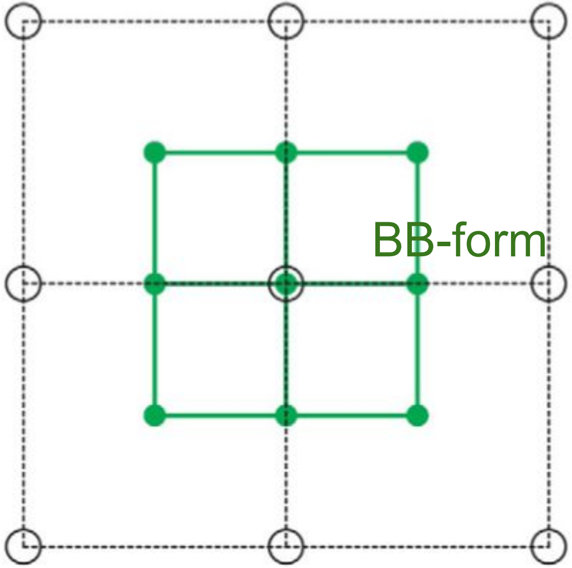
Recall: B-spline & BB-form

Interpolate corners





(a) BB-net and bi-2 patch



(b) bi-2 spline nets

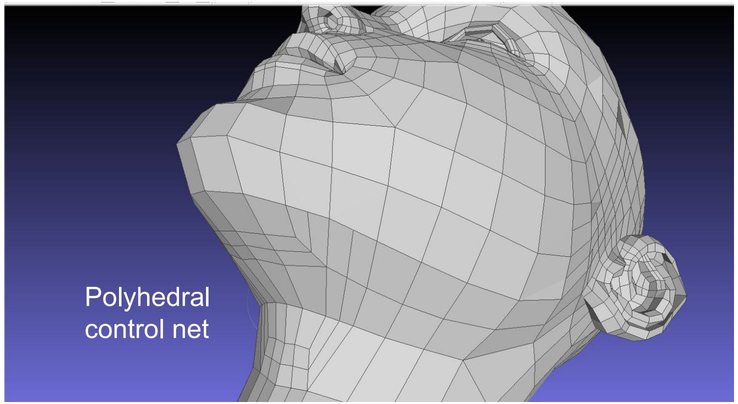
(uniform) B-spline

$\sum_i^n c_i \, b_i(t)$

$\sum_i^n c_i \, b_i(t)$

BB=Bernstein-Bezier

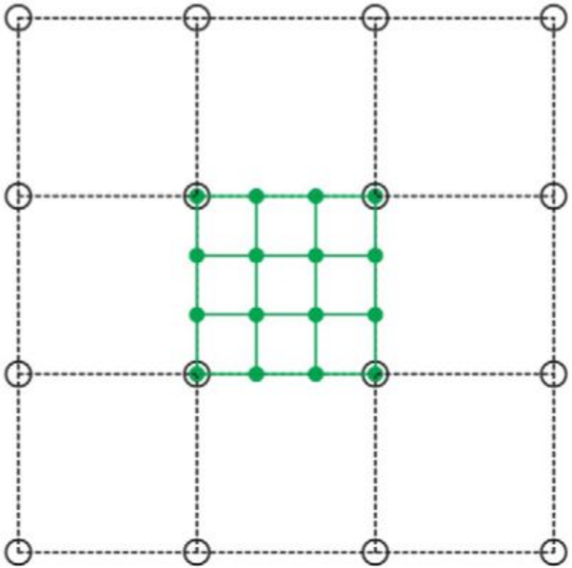
Polyhedral-net Splines (Geometry)



B-spline: built-in smoothess

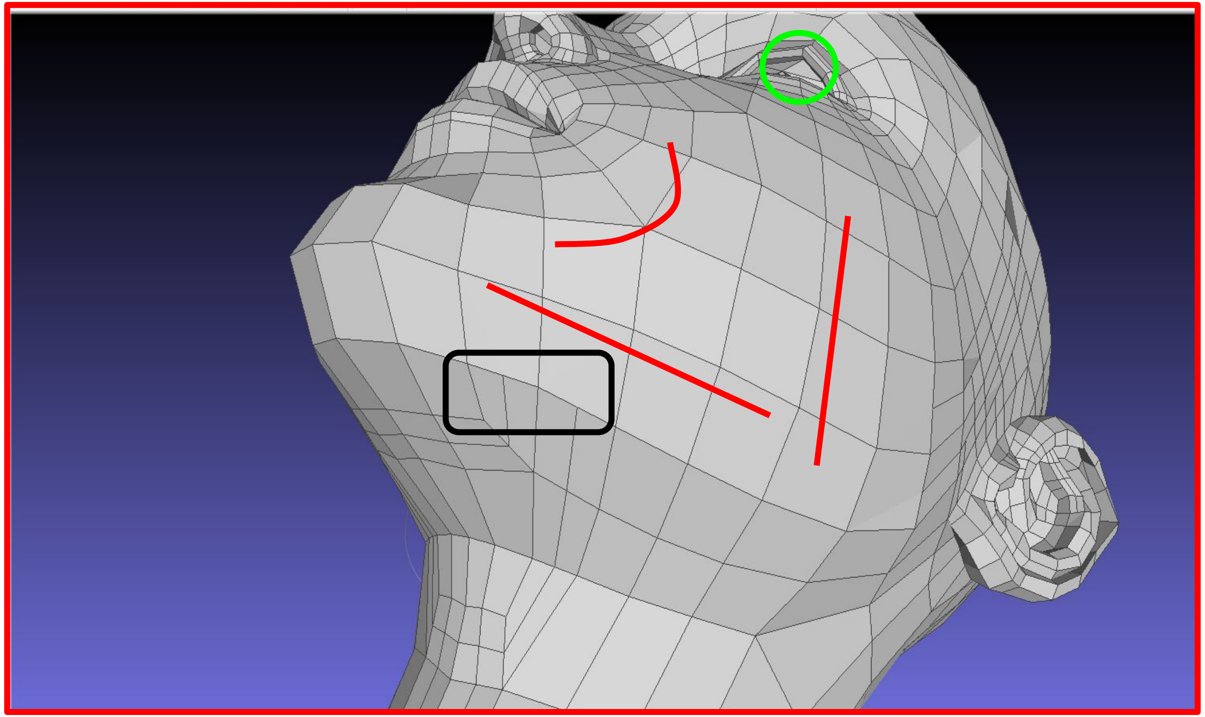
$$\sum_i^n c_i \, b_i(t)$$

$$\sum_i^n c_i \, b_i(t)$$

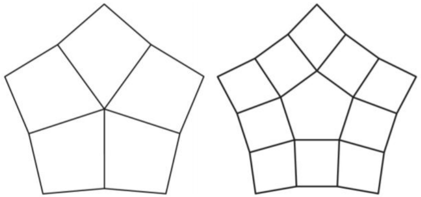


(c) bi-3 spline nets

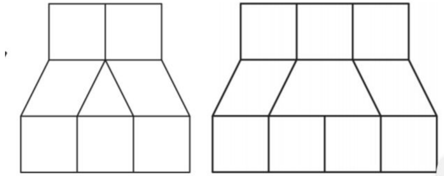
Polyhedral net patterns



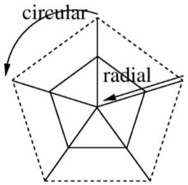
merge parameter directions



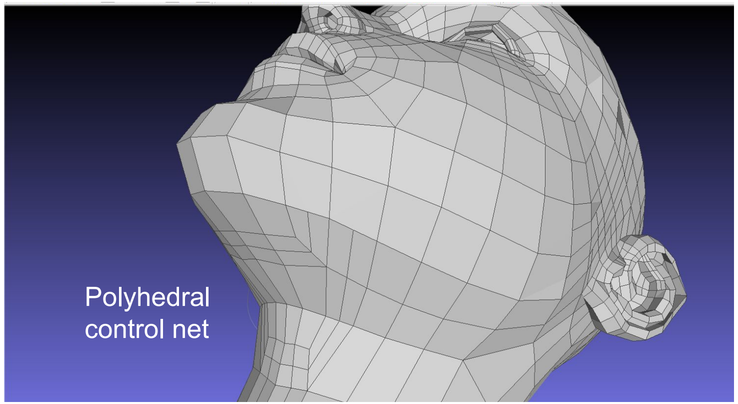
transition between coarse and fine meshes

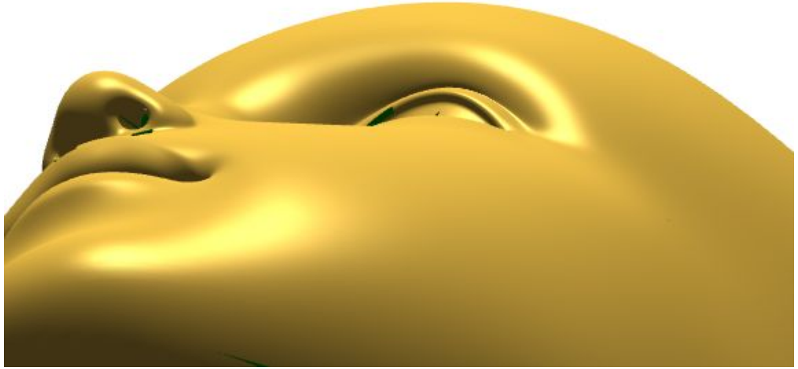


polar



Polyhedral-net Splines

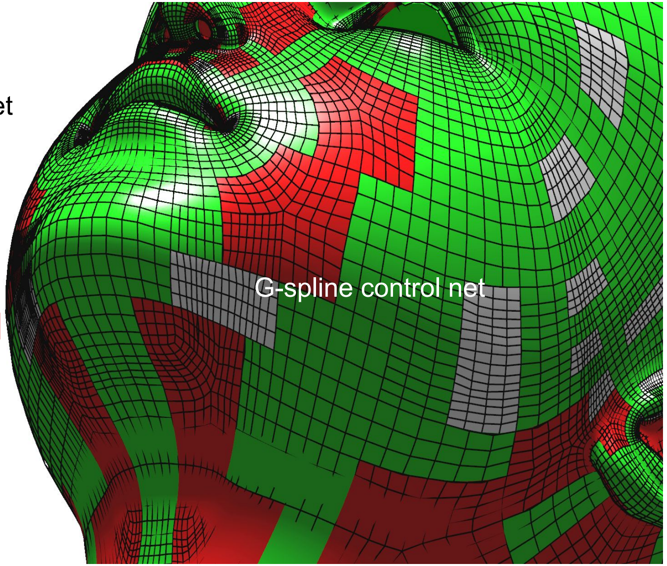




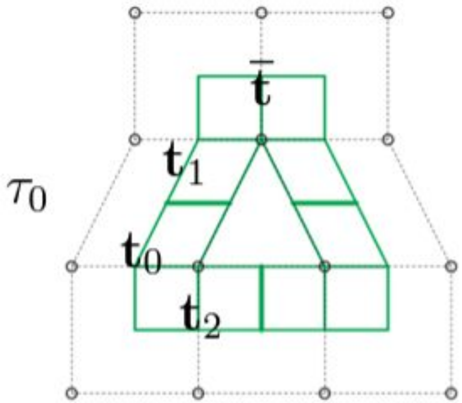
PnS3

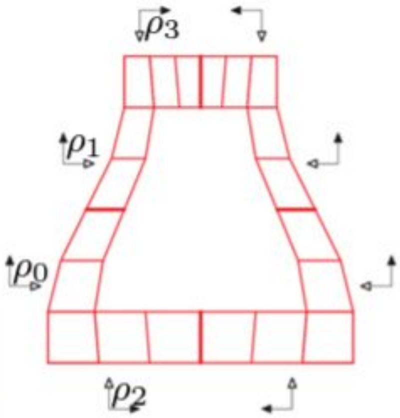
Bezier control net

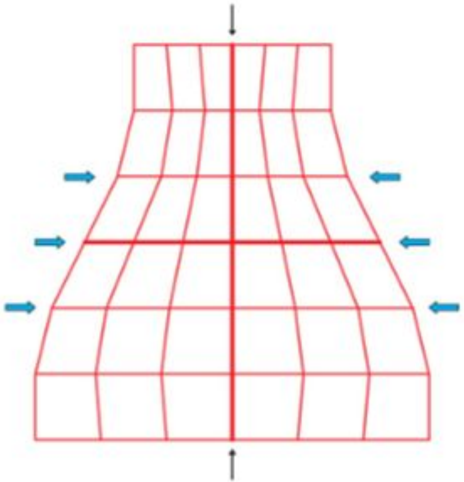
Bi-3 spline
T-junction
multi-sided



Tensor borders (Hermite data, ribbons,...)







config	$\alpha(u)$	$[\ldots, a_j(u), \ldots]$	$[\ldots, \beta_j, \ldots]$
τ_0	$1 - \frac{u}{2}$	$[\alpha(\frac{u}{2}), \alpha(\frac{1}{2} + \frac{u}{2}), 1, 1]$	$[0, 0, -1, 2]/4$
τ_1	$1 - \frac{u}{3}$	$[\alpha(\frac{u}{2}), \alpha(\frac{1}{2} + \frac{u}{2}), 1, 1, 1, 1]$	$[0, 0, -2, 2, 3, -3]/12$
τ_2	$1 - \frac{u}{3}$	see text above Table 1	$[-2, 2, -2, 2, 3, -3, 3, -3]/18$

BB-coefficients of reparameterization

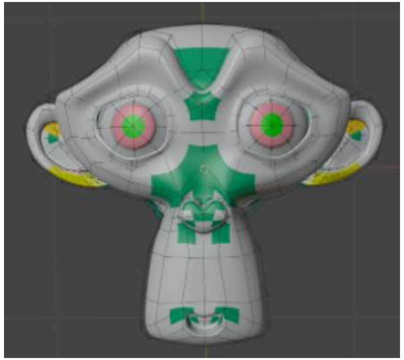
Table 1: Reparameterizations (change of variables) $\rho_j(u, v) := (u + b_j(u)v, a_j(u)v)$ as indexed in Fig. 4c.

Polyhedral-net Splines (PnS2)

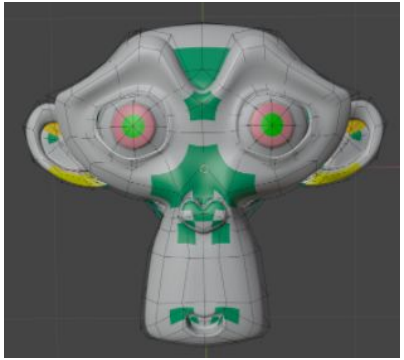
➢ Blender Add-on

Disclaimer:

- Blender offers only uniform bi-3 splines!
- Blender splines: Slow initialization!
- Blender renders incorrectly: evaluates lines, forms quads, uses approximate normals of quads —> export as iges and use FreeCAD



Polyhedral-net Splines (PnS2)



- Blender Add-on
  - Blender offers only uniform bi-3 splines!
  - Blender splines: Slow initialization!
  - Blender renders incorrectly: evaluates lines, forms quads, uses approximate normals of quads
- **AMC ToMS**: C++ Library,
  - Export to: IGES (freeCAD), bv (Bezierview)
  - Analysis add-on

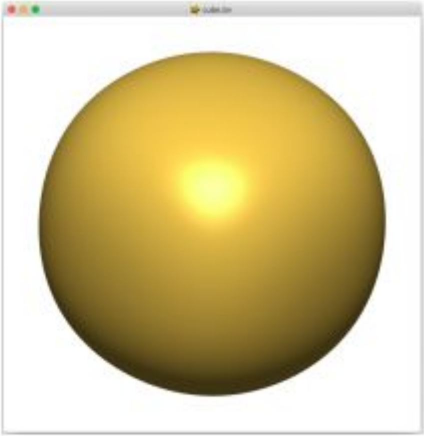
Polyhedral-net Splines


```
v 1.0 1.0 -1.0
v 1.0 -1.0 -1.0
v 1.0 1.0 1.0
v 1.0 -1.0 1.0
v -1.0 1.0 -1.0
v -1.0 -1.0 -1.0
v -1.0 1.0 1.0
v -1.0 -1.0 1.0
```

```
f 1 5 7 3
f 4 3 7 8
f 8 7 5 6
f 6 2 4 8
f 2 1 3 4
f 6 5 1 2
```

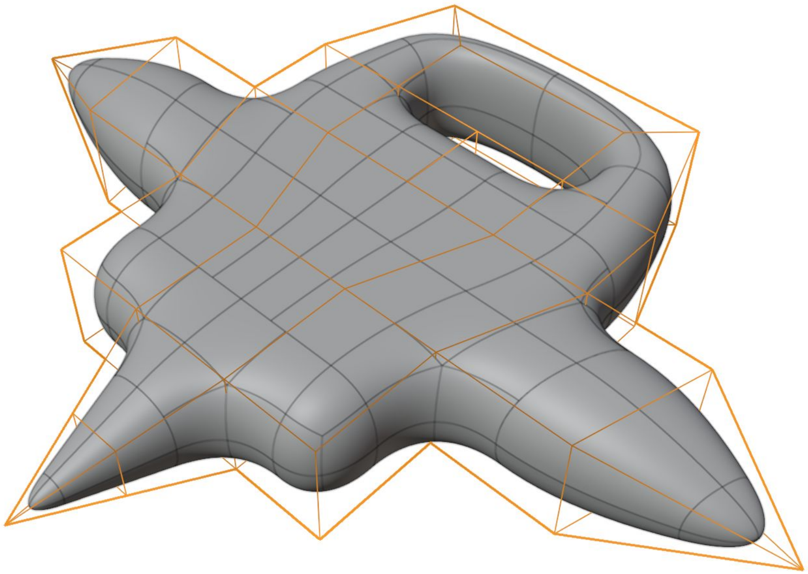
(a) Input: cube.obj

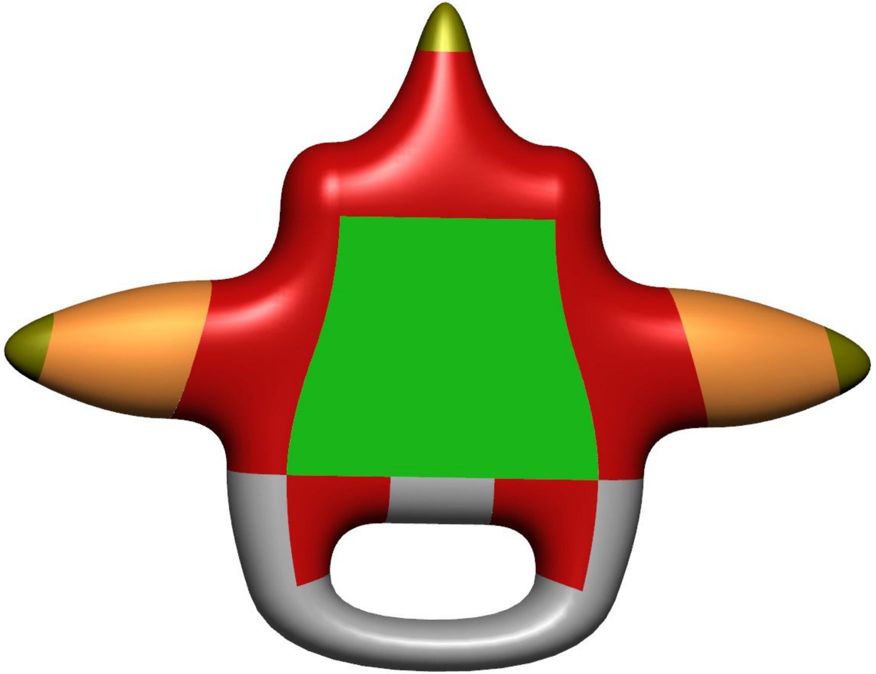
$\Longrightarrow$



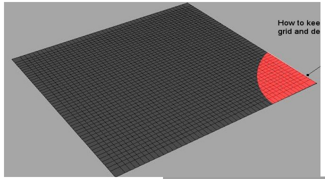
(b) surface

PnS shapes

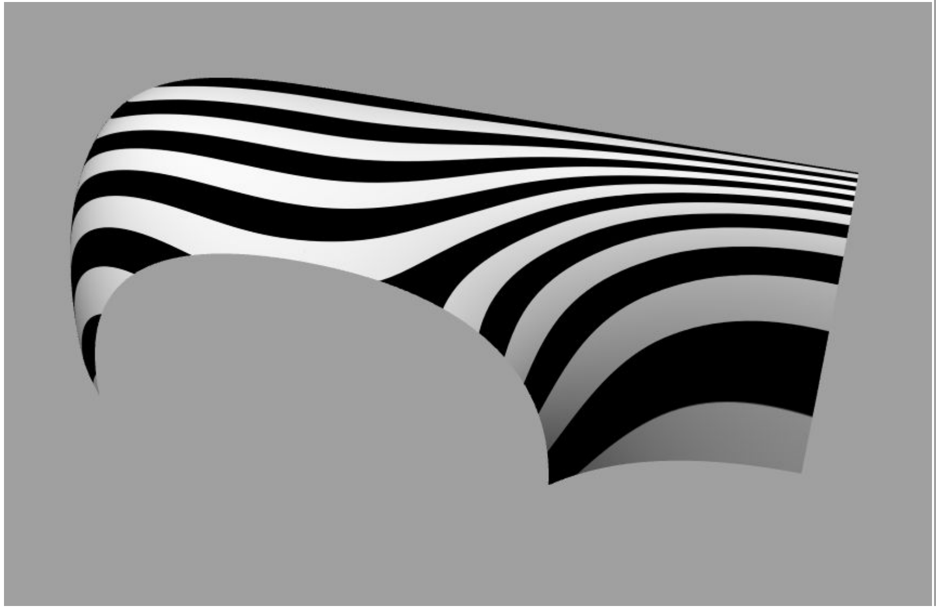


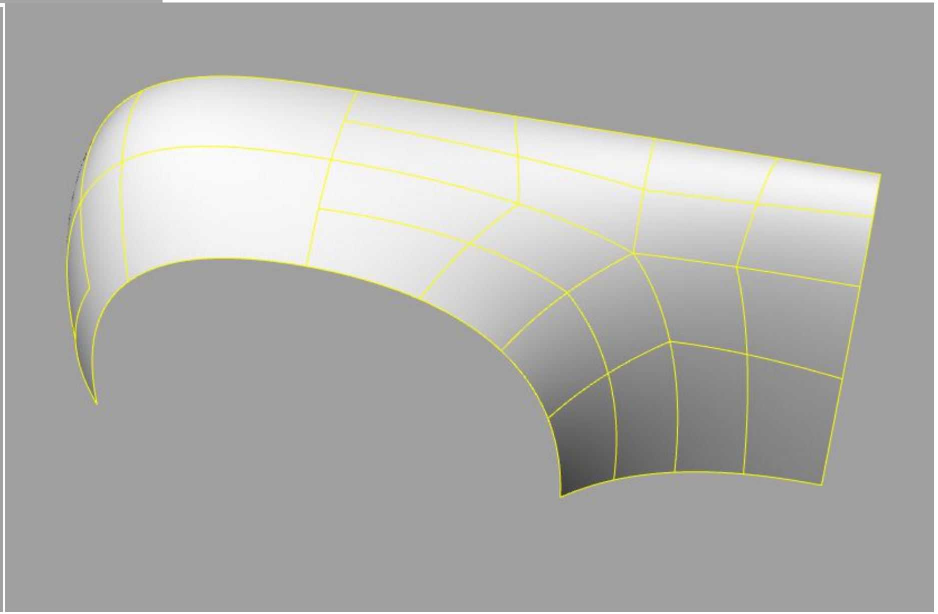


PnS avoids (one type of) trimming

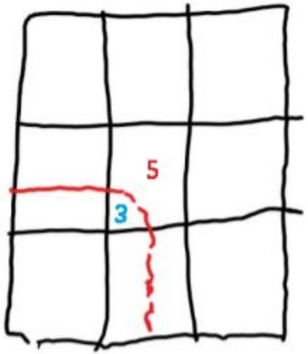


Trim → not exact (pixel dropout under zoom)

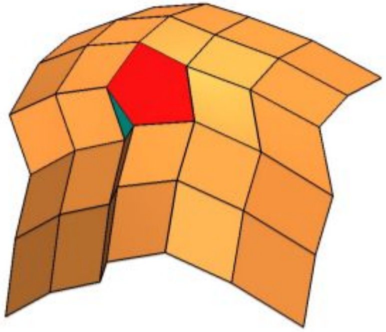




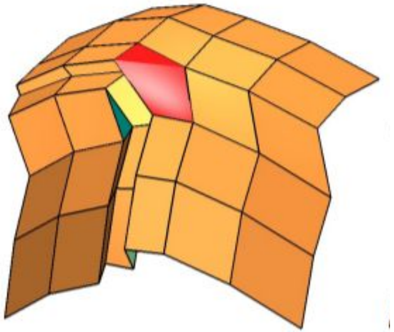
Polyhedral-net Splines: Alternative to (some types of) trimming



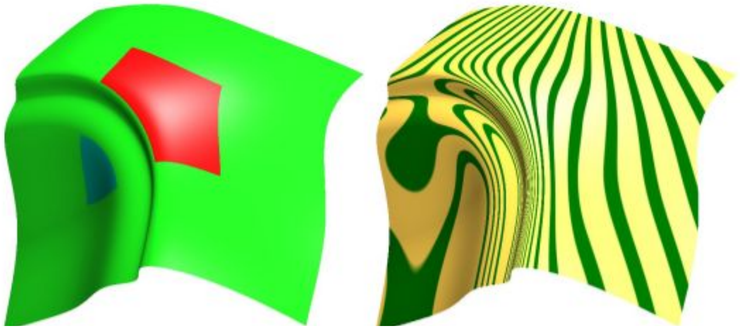
(a) sketch



(b) input with feature



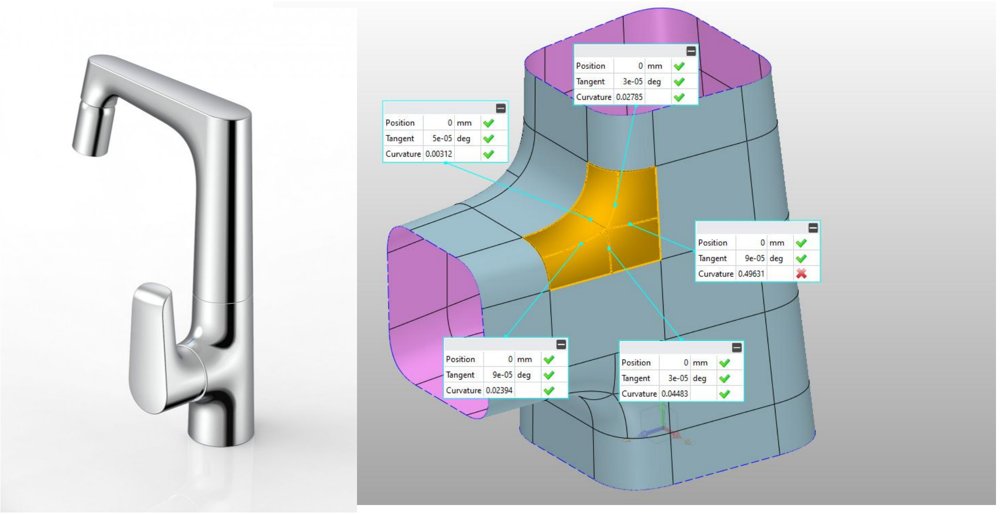
(c) groove inserted



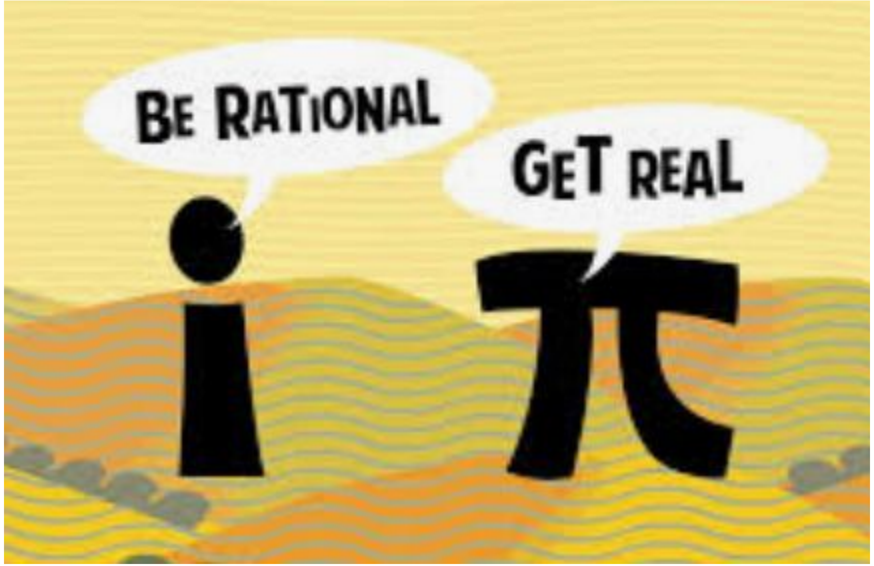
Designer: "Polyhedral Splines are better than Subdivision Surfaces"

PnS3

PnS2



Geometry



Analysis

A good idea: use splines for geometry and analysis

COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 44 (1984) 247–267
NORTH-HOLLAND

1983

SHAPE OPTIMAL DESIGN USING B-SPLINES

V. BRAIBANT and C. FLEURY
Aerospace Laboratory of the University of Liège, B-4000 Liège, Belgium

Received 28 January 1983
Revised manuscript received 21 December 1983

Shape optimal design of an elastic structure is formulated using a design element technique. It is shown that Bezier and B-spline curves, typical of the CAD philosophy, are well suited to the definition of design elements. Complex geometries can be described in a very compact way by a small

the Bezier or the B-spline techniques. Therefore the shape variables are no longer the positions of the 4, 8 or 12 nodes of an isoparametric two-dimensional element, but the points which control two families of curves whose Cartesian product defines the design element.

A good idea: use splines for geometry and analysis

COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 44 (1984) 247–267
NORTH-HOLLAND

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Received 28 January 1983
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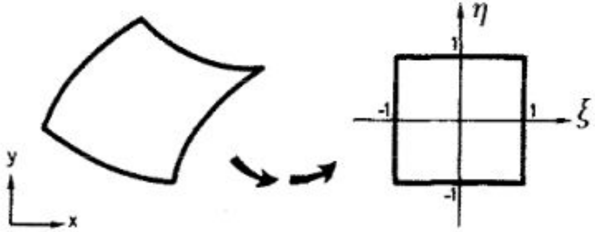
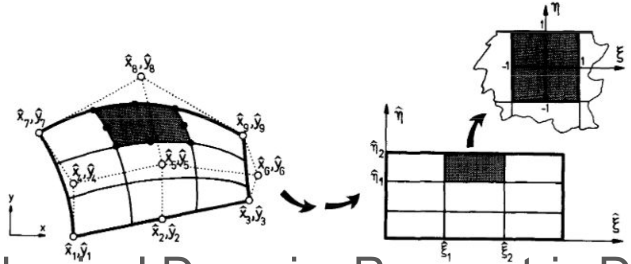
in which u_i and v_i are the nodal displacements. The formulation is isoparametric. In the subsequent calculus, the derivatives of the shape functions N_i with respect to the structural coordinates x, y will be needed:

$$\begin{bmatrix} N_{i,x} \\ N_{i,y} \end{bmatrix} = [J_0^{-1}] \begin{bmatrix} N_{i,\xi} \\ N_{i,\eta} \end{bmatrix} \tag{3.14}$$



Physical Domain Parametric Domain



Physical Domain Parametric Domain

A good idea: use splines for geometry and analysis

Computers & Structures Vol. 48, No. 1, pp. 23–32, 1993
Printed in Great Britain.

0045-7949/93 $6.00 + 0.00

1992

ISOPARAMETRIC SPLINE FINITE STRIP FOR PLANE STRUCTURES

F. T. K. Au and Y. K. Cheung
Department of Civil and Structural Engineering, University of Hong Kong, Hong Kong

(*Received* 4 *May* 1992)

Abstract—The isoparametric spline finite strip methods for Mindlin plate bending, plane stress and plane strain problems are presented. The essence of the method lies in the use of uniform cubic *B*-spline curves in the modelling of the geometry and representation of the displacement field. In this paper, the general

A. Y. T. Leung and F. T. K. Au. Spline finite elements for beam and plate. *Comput. Struct.* **37**, 717–729 (1990).

A good idea: use splines for geometry and analysis

PnS Jorg Peters

Finite Elements in Analysis and Design 15 (1993) 11–34 11
Elsevier

FINEL 340

1993

The coupling of geometric descriptions and finite elements using NURBs – A study in shape optimiz

Uwe Schramm and Walter D. Pilkey

Department of Mechanical, Aerospace and Nuclear Engineering, University of Virginia, Charlottesville, VA 22903, USA

Abstract. This paper presents a geometry based approach for coupling CAD with finite element methods. Non uniform rational B-splines (NURBs), known from computer aided geometric design, are used to describe the shape of a structure. NURB curves and surfaces and Gordon surfaces defined by NURB curves are introduced for design description. These geometries are used directly for the mapping of finite elements and for parameterization for the optimization of the structural shape. The cross-sectional torsion problem is employed

COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 71 (1988) 99–116
NORTH-HOLLAND

SHAPE OPTIMAL DESIGN USING HIGH-ORDER ELEMENTS

Y.K. SHYY and C. FLEURY
Mechanical, Aerospace and Nuclear Engineering Department, University of California at Los Angeles, Los Angeles, CA 90024, U.S.A.

and

K. IZADPANAH
The MacNeal-Schwendler Corporation, Los Angeles, CA 90041, U.S.A.

Received 17 December 1987

A good idea: use splines for geometry and analysis

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Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135–4195

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

2004

Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement

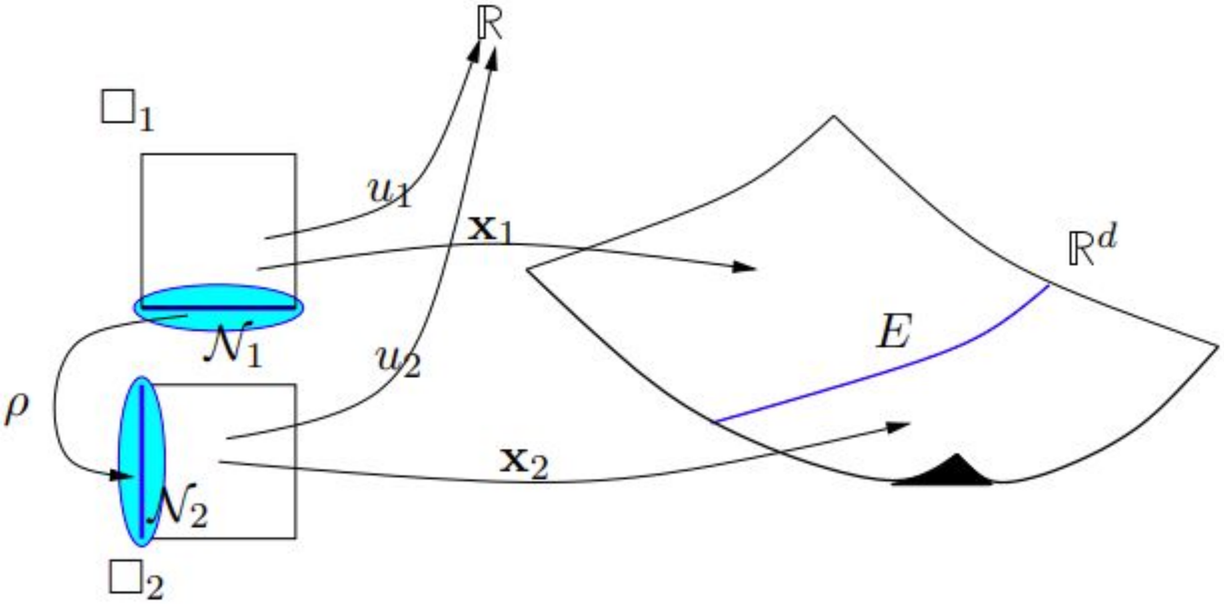
T.J.R. Hughes *, J.A. Cottrell, Y. Bazilevs

Institute for Computational Engineering and Sciences, The University of Texas at Austin, 201 East 24th Street, 1 University Station C0200, Austin, TX 78712-0027, United States

Received 28 September 2004; accepted 20 October 2004

Higher order isoparametric analysis (IGA)

Jorg Peters



Parametric Domain Physical Domain

Standard Galerkin weighted by first fundamental form

Jorg Peters

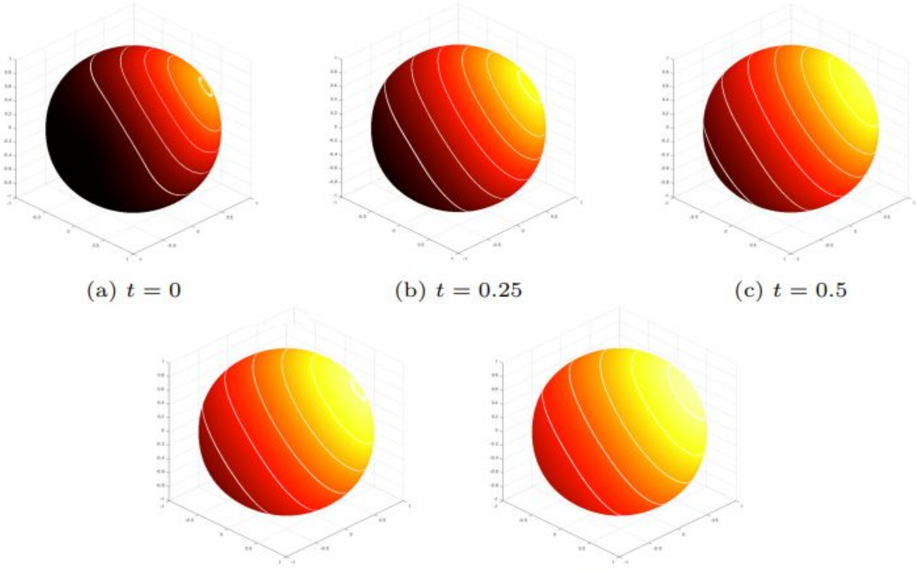
$$K_{ij} := \sum_\alpha \int_\square (\nabla\phi_i)^{\mathtt{t}} (J_\alpha^{\mathtt{t}} J_\alpha)^{-1} (\nabla\phi_j)\ \mathrm{J}\,\mathrm{d}\square, \quad \mathbf{f}_i := \sum_\alpha \int_\square f\phi^i\ \mathrm{J}\,\mathrm{d}\square.$$

1st fundamental form

$$J_\alpha := \nabla_{\mathbf{s}}\mathbf{x}_\alpha = \begin{vmatrix} \partial_s x & \partial_t x \\ \partial_s y & \partial_t y \end{vmatrix} \in \mathbb{R}^{2\times 2} \text{ and } \mathrm{J} := \sqrt{\det(J_\alpha^{\mathtt{t}} J_\alpha)} \in \mathbb{R},$$

Polyhedral-net Splines & Eng Analysis

(a) Input: cube.obj   (b) surface



(a) $t = 0$ (b) $t = 0.25$ (c) $t = 0.5$

(d) $t = 0.75$ (e) $t = 1$

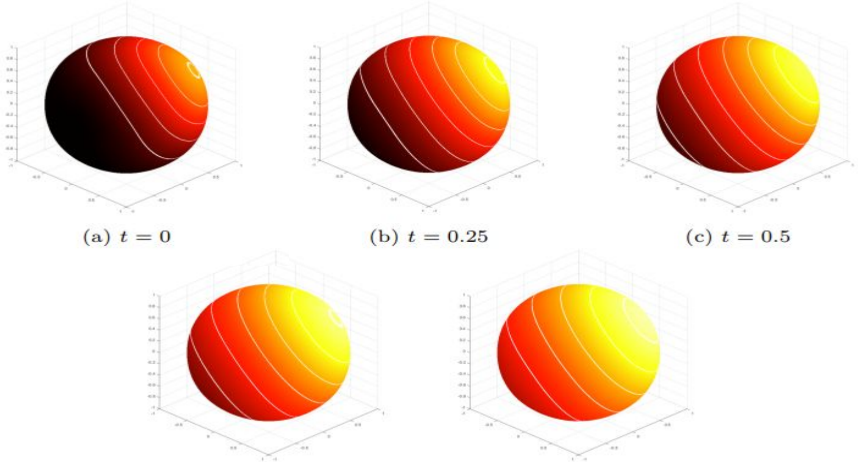
8 dof

control -net = dof (geometry, analysis function)

Geometry and Computing using Polyhedral-net Splines

Jorg Peters

8 dof vs



(a) $t = 0$ (b) $t = 0.25$ (c) $t = 0.5$ (d) $t = 0.75$ (e) $t = 1$

8x3 pieces
8x8 triangulation
degree 2 C0
volumetric
…

43008 dof

Geometry and Computing using Polyhedral-net Splines

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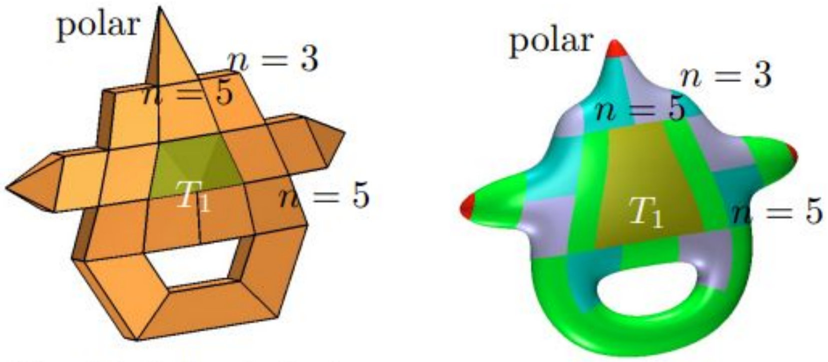
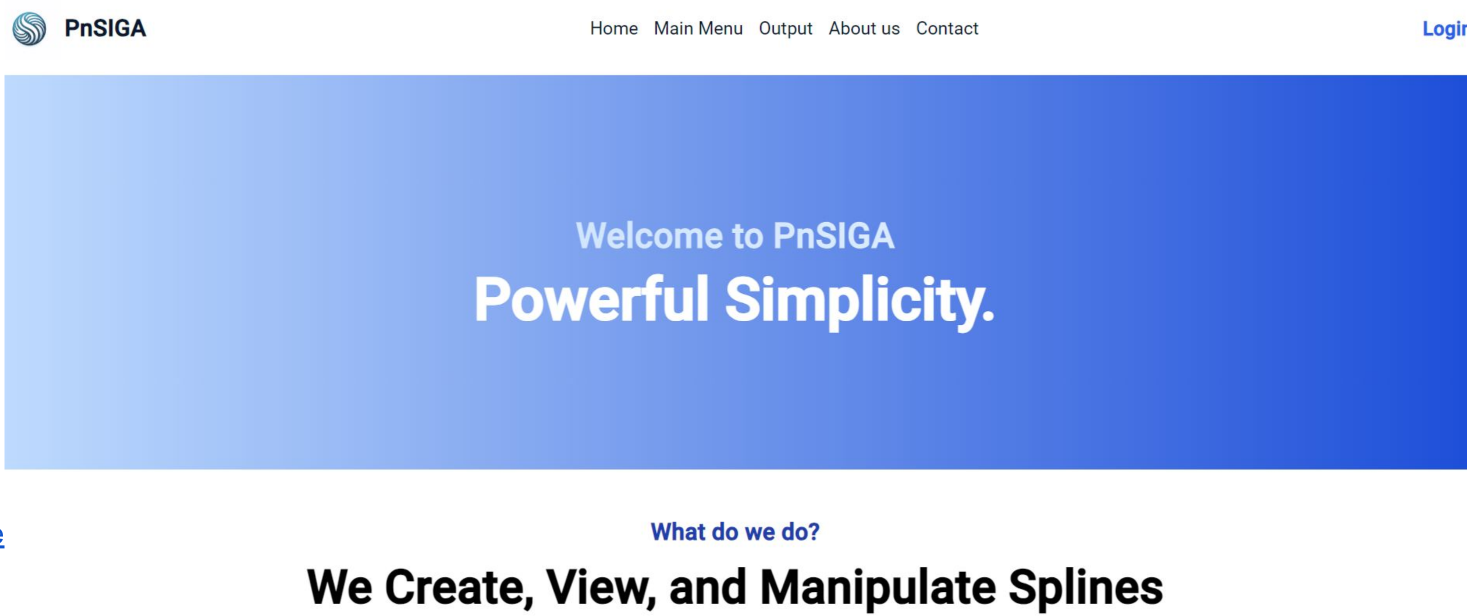


(a) polyhedral control net (b) spline patch layout

(a) $t = 0.5$ (b) $t = 0.75$ (c) $t = 1$

Geometry and Computing using Polyhedral-net Splines

Jorg Peters

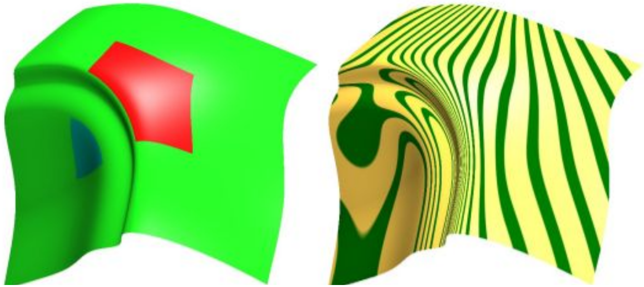


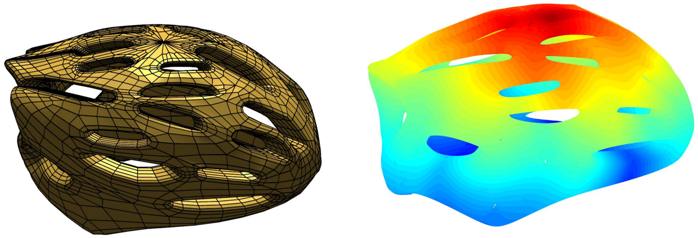
https://www.pnspline

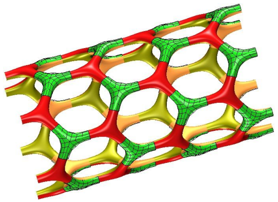
Polyhedral-net Splines for Geometry and Analysis

Jorg Peters

Blender Add-on **AMC ToMS** 2023: C++ PnS2 PnS3 **PnS_IGA**







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Polyhedral-net Surfaces for Geometry & Analysis

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Engineering analysis should match an underlying designed shape and not restrict the quality of the shape. I.e. one would like finite elements matched to a geometric space optimized for generically good shape.
Since the 1980s, classic tensor-product splines have been used both to define good shape geometry and analysis functions (finite elements) on the geometry.

Polyhedral-net splines (PnS) generalize tensor-product splines by allowing additional control net patterns required for free-form surfaces: *isotropic patterns*, such as n quads surrounding a vertex, an n-gon surrounded by quads, polar configurations where many triangles join; and *preferred direction pattern*s, that adjust parameter line density, such as T-junctions.
PnS2 generalize C1 bi-2 splines, generate C1 surfaces and can be output bi-3 Bezier pieces. There are two instances of PnS2 in the public domain: a Blender add-on and a ToMS distribution with output in several formats.
PnS3 generalize C2 bi-3 splines for high-end design.

PnS generalize the use of higher-order isoparametric approach from tensor-product splines. A web interface offers solving elliptic PDEs on PnS2 surfaces and using PnS2 finite elements.